Problem-Solving Strategies

:: Problem-Solving Strategies 1

BEHIND THE SCENES ::
BACKGROUND INFORMATION

Students will learn to use multiple problem-solving strategies.

1. Vocabulary Term:
   - **Equation:** A mathematical sentence that uses an equal sign to show that two quantities are equal.
2. Materials:
   - student math journals
   - pencils/pens
   - chalkboard/chalk
   - copies of Activity Sheets: Guided Practice Activity Sheet and Independent Practice Activity Sheet

SETTING THE STAGE :: *approximately 10 minutes*
OPENING ACTIVITY

1. On the chalkboard or overhead projector, write the following:

   Tropical Breeze:
   3 ounces orange juice
   5 ounces cranberry juice

2. Say, "The recipe I've just written is for a Tropical Breeze, my favorite juice drink. If I have 96 ounces of orange juice, how much cranberry juice would I need to add? In your math journal, write some ideas on how you would solve this problem. Then, discuss these ideas with a partner."
3. Walk around the room as students discuss the problem. Notice the different approaches they use. After 2 minutes, stop the partner discussions and select 3 or 4 students who used varied methods to share their ideas with the class.

4. On the chalkboard or overhead projector, keep a list of the strategies that students suggest. Possible strategies might include:
   - Use an Equation - a proportion of the form $\frac{3}{5} = \frac{96}{x}$
   - Make a Chart - list corresponding orange juice and cranberry juice quantities in the ratio of 3 to 5
   - Guess and Check - test values to see if the ratio is 3 to 5; if not, revise, and test again
   - Draw a Diagram - a picture showing 3 squares of orange juice for every 5 squares of cranberry juice could be repeated until there are 96 squares of orange juice
   - Work Backwards - the orange juice can make $96 \div 3$, or 32, servings of Tropical Breeze, so 32×5, or 180 ounces of cranberry juice must be added

   KAPtip You may wish to create a classroom display titled *Problem Solving Strategies We Know and Love*. As student suggestions are given in step 4 above, they can be used to start the list. Then, as students encounter additional strategies, they can be added, as well.

5. Say, "Nice job! That's a great list! As you can see, there are lots of ways to solve this problem. Today, we're going to look at several different problem solving-strategies that you can use."

DRESS REHEARSAL :: *approximately 12 minutes*

INSTRUCTION & GUIDED PRACTICE

1. Hand out the Guided Practice Activity Sheets.
2. Read the directions aloud with the class. Explain to the students that they will be solving the same problem in five different ways.

   KAPtip The final step in any problem-solving plan includes checking the answer. Because we are solving one problem in five different ways here, we will save the check until the end. You may wish to stress checking the solution as part of each method.

3. Select a student volunteer to read the problem aloud.
4. Ask, "Without working through the problem, who wants to take a guess at how many 2-point baskets Kobe scored?"
5. Call on a student to suggest a number. *Because Kobe made a total of 29 baskets, the suggestion should be from 0 to 29.*

6. In the Guess and Check area on the Activity Sheet, have all students write the number of 2-point shots that were suggested.

7. Based on the suggestions for the number of 2-point shots made, ask the class how many 3-point shots must have been made. *The number of 3-point shots is 29 minus the number of 2-point shots. If a student suggested that Kobe made 15 shots worth 2 points each, then he must have made 29 − 15, or 14 shots worth 3 points each.*

8. On their Activity Sheets, have all students write the corresponding number of 3-point shots.

9. With the class, determine the total number of points based on the suggestions. For example, with 15 shots worth 2 points and 14 shots worth 3 points, Kobe would have scored a total of 15(2) + 14(3), or 72 points. Have all students write the corresponding total score on their Activity Sheets.

10. If the total is not 63 points, have students revise the guess and check again. Continue until the correct answer is found. *Kobe made 24 of the 2-point shots and 5 of the 3-point shots.*

11. Say, "The problem-solving strategy we just used is called Guess and Check. The name says it all: make a guess, and check to see if it works. If it doesn't, revise your guess, and check again." Allow students to ask questions about the Guess and Check strategy. If there are no questions, move on to the next problem-solving strategy.

KAPtip Some students make educated (or lucky) guesses, and find this method easy to work with. Others struggle with this method because they don't know where to begin. Ask students to share how they determine which numbers to guess. Encourage students to question if their next guess should be higher or lower than their previous one.

12. Say, "The next problem-solving strategy we'll look at is called Make a Table." Explain that a table is just a way to organize information. On the chalkboard or overhead projector, fill in the first column as shown below, and have students fill in the first column on their Activity Sheets.

2-Point Shots	3-Point Shots	Total Points
15		
16		
17		
18		
19		
20		

KAPtip Consecutive numbers were deliberately chosen here so that the table can also be used later to demonstrate the next strategy, Look for a Pattern.

13. Call on students to complete the other two columns based on the information given in the problem. Students should complete the table on their Activity Sheets as follows:

2-Point Shots	3-Point Shots	Total Points
15	*14*	*72*
16	*13*	*71*
17	*12*	*70*
18	*11*	*69*
19	*10*	*68*
20	*9*	*67*
21	*8*	*66*
22	*7*	*65*
23	*6*	*64*
24	*5*	*63*

Our table shows us that Kobe made 24 of the 2-point shots, and 5 of the 3-point shots.

14. Say, "Making a table (or a chart, or list) is helpful in organizing information. In addition, a table can be used to help discover patterns, which is our next problem-solving strategy, Look for a Pattern."

15. Below the heading Look for a Pattern, have students list any patterns they notice in the table. Students should indicate that as numbers increase by 1 in the first column, numbers in the second and third columns decrease by 1.

16. Explain how the pattern could be used to find the answer. Say, "An organized table of information makes it easier to recognize patterns, so think about using a table, chart, or list when you encounter a pattern problem." *The pattern helps us to see that Kobe made 24 of the 2-point shots, and 5 of the 3-point shots.*

17. Answer any questions students have about the Make a Table and Look for a Pattern strategies.

18. Say, "The next strategy, Use an Equation, is one that you will use frequently." Work with students to write an equation (or a system of equations) that can be used to represent the situation. If x is used to represent the number of 2-point shots, then a single equation will result if $29 - x$ is used to represent the number of 3-point shots; alternatively, a system of equations will result if y is used to represent the number of 3-point shots. Both approaches, however, will yield a correct solution.

    Use an Equation in One Variable

    Let x represent the number of 2-point shots, and 29 − x represent the number of 3-point shots.

    $2x + 3(29 - x) = 63$

    $2x + 87 - 3x = 63$

    $-x = -24$

    $x = 24$

    $29 - x = 5$

    Use a System of Equations in Two Variables

    Let x represent the number of 2-point shots, and y represent the number of 3-point shots.

    $$\begin{cases} 2x + 3y = 63 \\ x + y = 29 \end{cases}$$

    Solve the second equation for x and use substitution.

    $2(29 - y) + 3y = 63$

    $58 - 2y + 3y = 63$

$y = 5$

$x = 29 - 5$

$x = 24$

OR

Solve the system by elimination

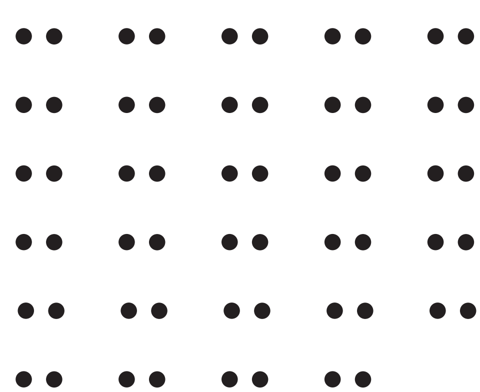
$$\begin{array}{r} 2x + 3y = 63 \\ \underline{-(2x + 2y = 58)} \\ y = 5 \\ 2x + 2(5) = 58 \\ x = 24 \end{array}$$

Kobe made 24 of the 2-point shots, and 5 of the 3-point shots.

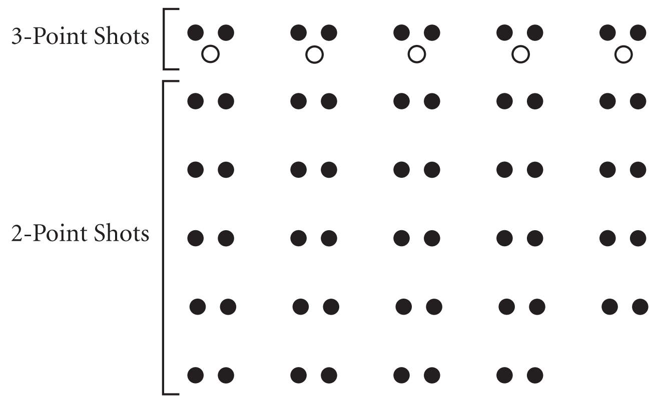
19. After solving the problem using an equation, ask the class, "Of the four strategies so far, Guess and Check, Make a Table, Look for a Pattern, and Use an Equation, which one do you like best?" Allow students to share their thoughts, but don't allow the discussion to last more than a minute or two. Conclude the discussion by emphasizing that more than one strategy may lead to a correct solution, and students should feel free to use whichever strategies they find most helpful. Students do not need to agree on which one they like best.

KAPtip Some students might indicate that they like to use a combination of strategies to solve some problems.

20. Say, "Let's look at one last strategy, called Draw a Diagram. For this strategy, you simply draw a picture that will help to solve the problem. For example, we can draw a diagram showing the number of shots that Kobe made." On the chalkboard or overhead projector, draw the following diagram:



21. Say, "Each dot in this diagram represents 1 point. Since there are 29 pairs of dots, this means that Kobe made 29 shots worth 2 points each. But, there are only 58 dots in this diagram, and we know that Kobe score 63 points, so we need 5 more dots." As shown below, update the diagram to include 5 additional dots. "If we add 5 dots to the diagram, we see that Kobe made 5 shots worth 3 points each, and 24 shots worth 2 points each."



22. Remind students that diagrams are very useful, especially when solving geometry and measurement problems. Answer any questions about the Draw a Diagram strategy.

23. Say, "We have now solved this problem using five different problem-solving strategies. In each case, we came up with the fact that Kobe made 24 of the 2-point shots, and 5 of the 3-point shots. Now, let's take a moment to check that we have answered the original question asked." Lead students through the check process by asking the following questions:

   - What does the question ask? *How many 3-point shots did Kobe make? How many 2-point shots did he make?*
   - Which number answers this question? *5;24*
   - What did you do to check that this answer is correct? *answers will vary*

24. Briefly review each of the five strategies with students. As you state the name of each of the five strategies, call on students to explain the strategy in one sentence.

25. Give students 2 minutes to record each of the five strategies in their math journals, along with a description of each.

SHOW TIME! :: approximately 18 minutes

INDEPENDENT PRACTICE

1. Distribute the Independent Practice Activity Sheets.
2. Read the directions aloud with students. Explain that they are to solve all five problems in this activity using a different strategy for each. To avoid using the same strategy more than once, suggest that students decide which one to use for each problem before solving.
3. Arrange students into pairs to complete the problems on the Activity Sheet.
4. After 10 minutes, combine pairs of students to form groups of four. Each group should review the strategies they used to find the solutions to the problems.
5. Ask each group to present a solution to one of the problems.

Independent Practice Activity Sheet Answers:

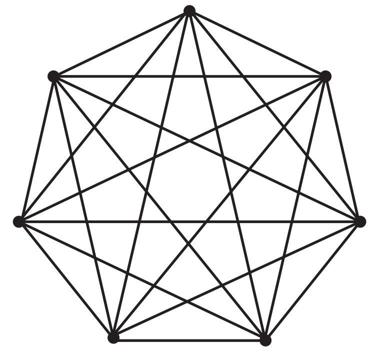
Note that many of the problems on the Activity Sheet could be solved with a variety of strategies, so several strategies are described below. Students, however, may use others.

1) *Look for a Pattern: If there were only two girls, there would be just 1 high-five. For three girls, 3 high-fives; for four girls, 6 high-fives. These numbers begin the pattern 1, 3, 6, 10, 15, 21, and so on. For seven girls, there are 21 high-fives.*

   Make a Table: The pattern may be more easily seen if the information is organized into a table.

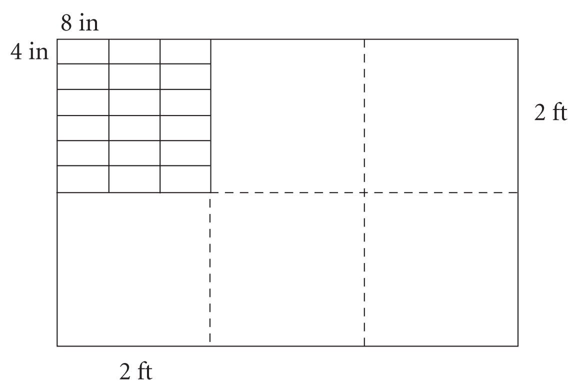
Number of Girls	Number of High-Fives
1	0
2	1
3	3
4	6
5	10
6	15
7	21

Draw a Diagram: To show the number of high fives, connect each of seven dots with every other dot. Each line represents a high-five, and the total number of lines, 21, is the total number of high-fives.



Check: The question asked me to find the number of high-fives, there were 21 high-fives. The question was answered.

2) *Draw a Diagram: As shown below, 18 tiles will cover a 2 feet by 2 feet section of the patio, and the patio has 6 of these sized sections. 18 × 6 = 108 tiles*



Check: The question is asking how many tiles will be used to cover the entire patio. We determined that 108 tiles would be used. We can check the answer by looking at the area of one tile, 32 in^2, (4 in × 8 in) and the area of the entire patio, 3,456 in^2 (48 in × 72 in). Finally, multiplying the area of one tile, 32 in^2, by 108 yields 3,456 in^2, which is equal to the area of the entire patio.

3) *Guess and Check: If all of the coins were dimes, Yemi would have $1.80. That's too much. If there were 12 dimes and 6 nickels, Yemi would have $1.50. Still too much. If there were 10 dimes and 8 nickels, Yemi would have $1.40. If there were 8 dimes and 10 nickels, Yemi would have $1.30.*

Make a Table: The information can be organized in a table as shown below:

Dimes	Nickels	Value
18	0	$1.80
17	1	$1.75
16	2	$1.70
15	3	$1.65
14	4	$1.60
13	5	$1.55
12	6	$1.50
11	7	$1.45
10	8	$1.40
9	9	$1.35
8	10	$1.30

Look for a Pattern: As the table above illustrates, when the number of dimes decreases by 1, the value decreases by 5¢. Hence, when there are 18 dimes, the value needs to decrease by 50¢, which is equal to 10 × 5¢. Consequently, the number of dimes must be decreased by 10, so that there are 8 dimes (and 10 nickels).

Use an Equation: Let d represent the number of dimes, and let n represent the number of nickels. Then $10d + 5n = 130$, and $d + n = 18$. This system yields $d = 8$, and $n = 100$.

Check: We are asked to find the number of nickels that Yemi has. Ten nickels gives Yemi $0.50, and 8 dimes equals $0.80, for a total of $1.30.

4) Use an Equation: Since an equation with a variable is given, solve for n.

$6 = \frac{n}{1 + \frac{2}{3}}$

$6 = n \div \frac{5}{3}$

$6 \times \frac{5}{3} = n$

$n = 10$

Guess and Check: Although it is not an obvious strategy, Guess and Check works well for this problem. First, simplify the right side to $n \div \frac{5}{3}$. Then, plug in values for n, and see if the right side of the equation is equal to 6.

- Although not a guess, note that $n = 0$ makes the right side equal to 0, which is too low.
- Try $n = 5$. That yields $5 \div \frac{5}{3}$, or 3. Again, this is too low, but note that it is exactly half of what is required in the problem.
- Try $n = 10$. That gives $10 \div \frac{5}{3}$, or 6.

Check: The question asks for the value of n. We can substitute 10 into the equation to check the results. $6 \stackrel{?}{=} \frac{10}{1 + \frac{2}{3}}$; $6 = 6$

5) Make a Table: The following table lists all of the numbers between 100 and 200. The numbers shown in bold are palindromes. As you can see, there are 10 in all.

101	**111**	**121**	**131**	**141**	**151**	**161**	**171**	**181**	**191**
102	112	122	132	142	152	162	172	182	192
103	113	123	133	143	153	163	173	183	193
104	114	124	134	144	154	164	174	184	194
105	115	125	135	145	155	165	175	185	195
106	116	126	136	146	156	166	176	186	196
107	117	127	137	147	157	167	177	187	197
108	118	128	138	148	158	168	178	188	198
109	119	129	139	149	159	169	179	189	199
110	120	130	140	150	160	170	180	190	--

Look for a Pattern: From the examples given in the problem, the first and last digits of a palindrome appear to be the same. This makes sense, since a palindrome is a number that reads exactly the same forward and backward. If a number is between 100 and 200, the first digit must be 1, so its last digit must also be 1. That yields numbers of the pattern 1-1, and any digit 0-9 can fill the blank. Consequently, there are 10 palindromes between 100 and 200.

Check: The question asks for the number of palindromes. The list shows that we have found them all.

RAVE REVIEWS ::

ASSESSMENT

Throughout elementary school, students approach problems using a variety of methods. As they learn algebra and geometry, which often rely on more formal methods, they tend to lose (or forget about) other problem-solving strategies. From time to time, it is valuable to remind students that strategies such as Guess and Check and Draw a Diagram are useful and reliable and, in fact, may be more efficient than a more formal method.

In addition, students may be uncomfortable using strategies for problems that they learned to solve a different way. For instance, students have likely used only equations, not a diagram, to solve problems like the one on the Guided Practice Activity Sheet. Students may therefore resist the idea that other strategies are possible. By solving problems using a variety of strategies, students will eventually realize that it is valuable to know more than one method.

1. As a quick quiz, have students solve the same problem using two different problem-solving strategies.
2. Have students keep a math journal in which they write about their understanding of this topic. For this lesson, students can respond to the following prompt: A friend tells you that solving an equation is the only way to tackle a math problem. Do you agree? Why or why not?
3. Check students' math journals to be sure that they have correctly recorded the five problem-solving strategies that were covered in this lesson.
4. Did each student successfully complete the Independent Practice Activity Sheet? Were the various problem-solving strategies correctly applied? Were the answers correct?
5. Observe and evaluate students' participation in the pair and group discussions. You may wish to use the Kaplan 4-Point Rubric (on next page) to score students in three areas: Solution Process, Presentation, and Understanding.

	Solution Process	Presentation	Understanding
1 Unsatisfactory	Not engaged in group's solution of the problem(s). Does not contribute to group effort at all.	Does not participate in the group presentation.	When asked, is unable to summarize the group's solution to the problem(s).
2 Developing	Is attentive while group solves problem(s) and contributes minimally to group effort.	Participates minimally in the group presentation.	Is able to explain parts of the group's solution, but indicates an incomplete understanding of how the steps in the solution led to the correct answer.
3 Mastery	Participates in group's solution of the problem(s). Contributes significantly by suggesting strategies, performing computation, and/or recording work.	Participates in the group presentation in two of the following ways: explaining strategies; showing calculations; drawing accompanying charts, lists, or diagrams; and/or answering questions from peers.	Is able to explain the group's solution completely. Explanation indicates an understanding of the mathematics behind the solution, with some gaps.
4 Excellence	Contributes significantly by suggesting strategies, performing computation, and/or recording work. Cooperates in the group by listening to all suggestions, and helps to evaluate all solution methods before selecting an approach.	Cooperates with group members in presenting the solution. Contributes significantly in three or more of the following ways: explaining strategies; showing calculations; drawing accompanying charts, lists, or diagrams; and/or answering questions from peers.	Is able to explain multiple solutions to the problem. Explanation indicates a full understanding of the mathematics behind the solution, with no gaps.

ENCORE! ENCORE! ::

EXTENSION ACTIVITIES

1. If more time is available, students will gain a deeper conceptual understanding of using multiple problem-solving strategies by completing the SkillTivity lesson.
2. As you proceed through your regular curriculum, look for opportunities to have students use multiple problem-solving strategies so that they continue to practice applying these skills in various scenarios. When students work in groups, have them present at least two different solution methods for each problem they solve.
3. As suggested in the Opening Activity, create a classroom bulletin board display titled *Problem Solving Strategies We Know and Love.* As students encounter new strategies, add them to the list.

Name ______________________ Date ______________

:: Problem-Solving Strategies 1

GUIDED PRACTICE

Solve the following problem using each of the problem-solving strategies listed below.

Kobe scored 63 points in his last basketball game. He made a total of 29 shots, each of which was worth either 2 or 3 points. How many 2-point shots did he make? How many 3-point shots did he make?

Guess and Check

Make a Table

2-Point Shots	3-Point Shots	Total Points

Look for a Pattern

Name ______________________ Date ______________

GUIDED PRACTICE *(cont'd)*

Use an Equation

Draw a Diagram

Name ______________________________ Date ____________________

:: Problem-Solving Strategies 1

INDEPENDENT PRACTICE

Use a different problem-solving strategy to solve each of the following problems.

1. As each player on the basketball team's name was announced, they sprinted onto the court and high-fived their teammates whose names had already been called. If seven girls were announced in all, how many high-fives occurred?

Strategy used: ______________________________

2. To cover a patio that is 4 feet by 6 feet, Maria is using rectangular tiles that measure 8 inches by 4 inches. How many tiles will she need to cover the entire patio?

Strategy used: ______________________________

Name ______________________ Date ______________

INDEPENDENT PRACTICE *(cont'd)*

3. Yemi has 18 coins in her pocket, all of which are either nickels or dimes. If the total value of her coins is $1.30, how many nickels does she have?

Strategy used: ______________________

4. Solve for n: $6 = \dfrac{n}{1 + \frac{2}{3}}$

Strategy used: ______________________

5. A palindrome is a number that reads the same backwards and forwards, such as 55 and 393. How many palindromes are there between 100 and 200?

Strategy used: ______________________

:: Problem-Solving Strategies 1

BEHIND THE SCENES ::
BACKGROUND INFORMATION

Students will use multiple problem-solving strategies.

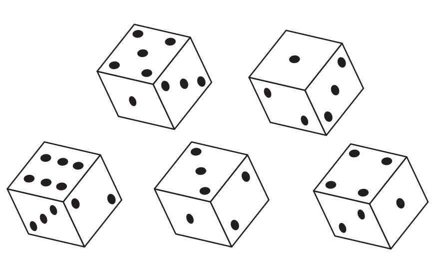
1. Students should already be familiar with the following vocabulary: **equation.**

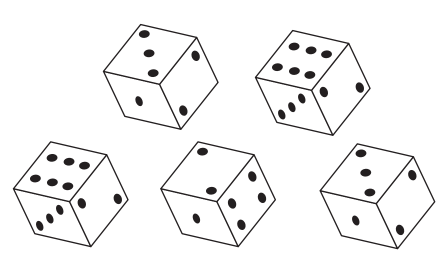
2. Materials:
   - student math journals
   - pencils/pens
   - chalkboard/chalk
   - five standard dice (the dice should have pips (dots), not numerals, on the six faces)
   - copies of Activity Sheets: Guided Practice Activity Sheet and Independent Practice Activity Sheet

SETTING THE STAGE :: *approximately 10 minutes*
OPENING ACTIVITY

1. As students enter the classroom, sit at your desk, rolling five dice. After each roll, say a number, based on the rules of *Petals Around The Rose*.

   KAPtip *Petals Around The Rose* is played by rolling five dice. Each roll has a unique answer, found by adding all of the *petals* around the *rose* that appear on dice with an odd number of pips. That is, there are two petals around the rose when a 3 is rolled, and there are four petals around the rose when a 5 is rolled. For the roll seen below left, the answer is 6 (4 + 2). For the roll seen below right, the answer is 4 (2 + 2). It is possible to have a roll showing 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, or 20 petals.





To play the game, roll the dice, and tell students the answer (the amount of petals around the rose). The object of the game is for students to determine the rule; that is, they are to figure out how you are arriving at the answer for each roll.

2. When students ask what you are doing, say, "This game is called *Petals Around The Rose*. The name of the game is very important. Each time I roll the dice, there is exactly one answer." Then, roll the dice and say the answer for that roll. Do not give students any other information; when they ask again, give the same response: "This game is called *Petals Around The Rose*. The name of the game is very important. Each time I roll the dice, there is exactly one answer."
3. Continue to roll the dice and report the answers until all students have seen several rolls.
4. Ask all students to be seated. Ask, "Has anyone figured out the rules for *Petals Around The Rose*?"
5. If any student claims to know the rules, test them by rolling the dice and having them state the answer. At this point, it is unlikely that a student will have figured out the rules.

   KAPtip A number of companies make transparent dice. When rolled, these dice will show the pips on an overhead projector. They are very useful for sharing the rolls with the entire class, and can be found at most educational supply stores.

6. Ask, "Is there anything that you'd like me to do that would help you figure out the rules?" Allow students to offer suggestions, and prompt them to offer three strategies:
   - **Make a chart to record the results.** It will help to have a record of many rolls. An organized list will make it easier to recognize patterns.
   - **Consider a simpler case.** By rolling fewer dice, it may be easier to identify the rules. In fact, the most helpful strategy is seeing what happens when just one die is rolled.
   - **Guess and check.** Students can test their conjectures by comparing their answers with the answers you provide.
7. Follow the suggestions made by students to help them determine the rules of the game. In particular, require students to keep a record of the rolls, and roll fewer than five dice to give students some simpler examples.
8. When a student figures out the rules, name them the *Ambassador of the Rose*. With this title, they are now allowed to roll the dice, but are not allowed to tell anyone the rules.
9. Allow the game to continue for up to 5 minutes. As each student figures out the rules, they then become the *Ambassador of the Rose,* and take over the dice.

10. Briefly review the strategies that were helpful to students in discovering the rules of the game. Then say, "You've applied a lot of good problem-solving techniques to determine the rules for *Petals Around The Rose*. Now let's apply similar problem-solving strategies to some math problems."

    Opening Activity Answers:

    Answers will vary.

DRESS REHEARSAL :: approximately 15 minutes

INSTRUCTION & GUIDED PRACTICE

1. Distribute the Guided Practice Activity Sheets.

2. Read the directions aloud with students. Explain to the class that the Activity Sheet contains five problems. Along with each problem, there is a description of how a student attempted to solve it. Point out that students are to first identify the strategy that was tried, and then solve the problem using the same strategy.

3. Indicate that there are nine different strategies to consider:

    - Consider a Simpler Case
    - Eliminate
    - Work Backwards
    - Make a Model
    - Guess and Check
    - Look for a Pattern
    - Make a Table, Chart, or List
    - Draw a Diagram
    - Use an Equation

    Any of these strategies may have been used in the attempted solutions.

4. Have students take notes in their math journals as you review the strategies that may not be familiar to them.

    - **Consider a Simpler Case:** In figuring out the rules for *Petals Around The Rose*, the strategy of considering what happens with fewer than five dice is an example of considering a simpler case. This strategy can be used any time a problem can be divided into smaller, more manageable pieces.
    - **Eliminate:** This strategy works well in partnership with the *Make a Table, Chart, or List* strategy. Once a list is made of all possibilities, those that do not meet the criteria of the problem should be discarded.
    - **Work Backwards:** This is a strategy to use when a final result is known. As an example, if we know that a student was left with 10 sticks of gum after giving a stick to each of 7 friends, working backwards we can see that he must have started with 17 sticks of gum.

- **Make a Model:** Using pennies, sticks, pieces of paper, or other objects can help make concrete sense of the problem.
- **Guess and Check:** Take a reasonable guess at the answer, and check. If the guess works, the answer is found; if it doesn't, revise the guess and check again. The *Guess and Check* strategy is most effective when the range of the answer is relatively small. If there is no way to make an estimate of the answer, this might not be the best strategy to use.
- **Look for a Pattern:** Consider how the numbers in the problem change. If they vary in a predictable way, the pattern can be extended to find the answer.
- **Make a Table, Chart, or List:** A table, chart, or list can be used to organize the information provided into a more manageable form. A table, chart, or list also makes it easier to apply each of the *Guess and Check*, *Look for a Pattern*, and *Eliminate* strategies, so they are often used in partnership.
- **Draw a Diagram:** A picture is worth a thousand words, and sometimes it is easier to see a solution than to calculate it. This strategy is particularly helpful with geometry problems that use words to describe a situation; often, drawing a diagram will lead to an immediate solution.
- **Use an Equation:** As the name implies, this strategy involves assigning variables, creating equations, and solving. Though valuable, this strategy tends to be overused. Especially when students study algebra, they often use this strategy when others might actually be more efficient. For instance, an equation can be used to solve the problem, "Three times a number is 14 more than the same number. What is the number?" but it might be easier to use the *Guess and Check* method here. It is valuable for students to learn when equations are the best or only solution, and when there are others they should consider.

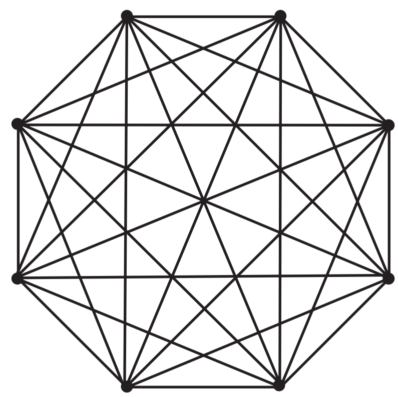
KAPtip Your students may already be familiar with several of these strategies. Briefly review those with which they are already familiar, but devote more time to the ones they haven't seen before.

5. Together with the class, review the Activity Sheet. For each question, call on a student to read the problem aloud. Call on another student to read the attempted solutions. Work with students to determine which strategy was used. Allow a brief discussion about the strategy for each problem, but do not let the discussion run too long. Finally, work with the class to solve the problem using the strategy identified.

Guided Practice Activity Sheet Answers:

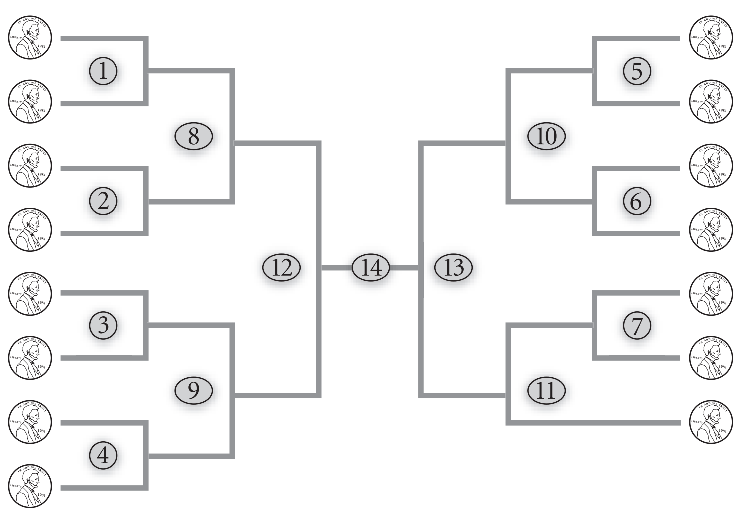
1) *Draw a Diagram*

   A diagram with 8 points, in which the points are connected with lines, can be used to represent the situation. As shown below, there are a total of 28 lines, so 28 games must have been played at the tournament with 8 teams.



2) *Make a Model*

   Using pennies (or other tokens) to represent the teams, a concrete model can be used to act out the tournament. The number of games necessary can be counted as the tournament proceeds.



3) *Guess and Check*

Sarah tried 20 yards by 60 yards, which has an area of 1,200 square yards. Since that's too small, try a larger width.

Try 30 yards for the width. Then, the length must be 3 × 30, or 90 yards, and the area would be 30 × 90, or 2,700 square yards. That's still too small, so try an even larger width.

Try 40 yards for the width. Then, the length must be 3 × 40, or 120 yards, and the area would be 40 × 120, or 4,800 square yards. Since that's the area given in the problem, an ultimate frisbee field must be 40 yards wide.

4) *Elimination*

The possible numbers here are the integers from 10 to 99. However, the third clue (less than 50) eliminates the integers between 50 and 99, so a list of integers from 10 through 49 can be used.

10	20	30	40
11	21	31	41
12	22	32	42
13	23	33	43
14	24	34	44
15	25	35	45
16	26	36	46
17	27	37	47
18	28	38	48
19	29	39	49

The second clue (odd) eliminates the first, third, fifth, seventh, and ninth rows, leaving the following:

11	21	31	41
13	23	33	43
15	25	35	45
17	27	37	47
19	29	39	49

The first clue (only 1, 3, 4, 5, and 7) then eliminates the second column (which contains the digit 2) and the fourth row (which contains the digit 9).

11	31	41
13	33	43
15	35	45
17	37	47

Finally, based on the last clue, the only number remaining in which the sum of the digits is greater than 10 is 47, so Jeremy's uniform number must be 47.

5) *Look for a Pattern*

Antonia noticed that the number of teams increased by 4 (from 8 to 12), and then increased by 6 (from 12 to 18). It seems, then, that the number of teams will increase by 8 next year, so there will be 18 + 8, or 26 teams in the league.

Another possible pattern is that the number of teams is equal to the previous year's number times 1.5:

$8 \times 1.5 = 12$

$12 \times 1.5 = 18$

Therefore, it is also reasonable to conclude that there will be $18 \times 1.5 = 27$ teams in the league next year.

SHOW TIME! :: approximately 15 minutes

INDEPENDENT PRACTICE

1. Distribute the Independent Practice Activity Sheets.
2. Read the directions aloud with the class. Be sure to review the problem-solving strategies listed on the Activity Sheet, and take a moment to briefly answer any questions students may have regarding the strategies listed.

   ✓KAPtip As questions are asked, answer for the benefit of the entire class if the question is relevant for all students. For more specific questions, you may choose to wait and answer them individually while students are working.

3. Say, "Solve the four problems using one of the strategies listed. Before solving the problem, however, talk with your group members about which strategy you think would be the most effective."
4. Assign students to groups of three to solve the four questions on the Activity Sheet.

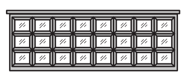
5. Circulate around the room as students solve the problems. Answer any questions and offer assistance as needed.

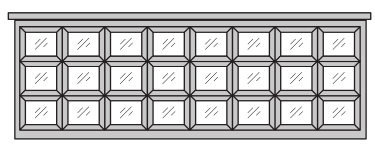
KAPtip Be careful not to offer too much assistance. Students only develop problem-solving skills by solving problems. By offering too much information, you may be denying students the opportunity to develop these skills on their own.

Independent Practice Activity Sheet Answers:

1) *Possible strategies: Draw a Diagram; Use an Equation; Look for a Pattern*

Draw a Diagram: A diagram can be used to help when setting up the proportion below.





Use an Equation: The proportion $\frac{56}{20} = \frac{x}{70}$*, where x represents the actual length, yields* $20x = 3{,}920$*, or* $x = 196$*.*

Look for a Pattern: As shown in the table below, the length and width of the model can be increased as necessary until the appropriate width is found. Multiplying both dimensions by the same amount keeps the proportion intact. The corresponding length will be the answer.

	Length	Width
Dimensions of model	56 inches or $4\frac{2}{3}$ feet	20 inches or $1\frac{2}{3}$ feet
Multiply by 12	56 feet	20 feet
Multiply second row by 2	112 feet	40 feet
Multiply second row by 3	168 feet	60 feet
Multiply second row by 3.5	196 feet	70 feet
Multiply second row by 4	224 feet	80 feet

2) Possible strategies: Make a Table, Chart, or List; Elimination

Make a Table, Chart, or List: A list can be made to identify the possible values. The sum of the first two numbers is 19 + 27, or 46, so the possible numbers are 47 through 60.

47 48 49 50 51 52 53 54 55 56 57 58 59 60

Elimination: From the list above, eliminate those numbers that do not meet the criteria. All of the even numbers can be removed, since the third number is odd. That leaves 47, 49, 51, 53, 55, 57, and 59. Of those, only 53 contains the digit 3.

3) Possible strategies: Use an Equation; Make a Table, Chart, or List; Guess and Check

Use an Equation: Let d represent the number of days at which the rentals are equal.

$46 + 23d = 18 + 30d$

$28 = 7d$

$d = 4$

The rental price would be the same if the cars were rented for 4 days.

Make a Table, Chart, or List: As shown below, a table can be used to compute the rental cost for each company:

Number of Days	Axis	Wertz
1	$69	$48
2	$92	$78
3	$115	$108
4	$138	$138
5	$161	$168

Guess and Check: By guessing numbers to represent the days rented, the answer can be found. As a first guess, try 5 days. For 5 days, the cost for Axis is $161, and the cost for Wertz is $168. Since Wertz has a higher daily rate, we know that the number of days must be less than 5.

Try 4 days. Axis is $161, and Wertz is $161, so the answer is 4 days.

4) Possible strategies: Use an Equation

An equation can be used to solve this problem. There are several equations that will work:

Equation #1: The proportion $\frac{60}{100} = \frac{18}{x}$ can be used to find the number of girls in the band. That yields $x = 12$, or 12 girls in the band. Half of them play percussion instruments, so there must be 6 girls in the percussion section.

Equation #2: If sixty percent are boys, then forty percent are girls. Consequently, the equation $\frac{60}{40} = \frac{18}{x}$ can be used to find the number of girls in the band. That yields $x = 12$, so there are 12 girls in the band, and must be 6 girls in the percussion section.

Equation #3: The equation $\frac{1}{3} \times 18 = x$ can be used to find the number of boys in the percussion section. This yields $x = 6$, so there are 6 boys in the percussion section. Further, we know that the ratio of boys to girls in the band is 60:40, or 3:2. Since one-third of the boys and one-half of the girls play percussion instruments, the ratio of boys to girls who play percussion is $\frac{3 \times \frac{1}{3}}{2 \times \frac{1}{2}}$, or 1:1. This implies that there are an equal number of boys and girls in the percussion section. Consequently, there must be 6 girls who play percussion instruments, since 6 boys play percussion instruments.

RAVE REVIEWS ::

ASSESSMENT

Throughout elementary school, students attack problems using a variety of methods. As they learn algebra and geometry, which often rely on more formal methods, they tend to lose (or forget about) other problem-solving strategies. From time to time, it is valuable to remind students that strategies such as *Guess and Check* and *Draw a Diagram* are useful and reliable and, in fact, may be more efficient than more formal methods.

In addition, students may be uncomfortable using strategies for problems that they learned to solve in a different way. Students may therefore resist the idea that other strategies are possible. By solving problems using a variety of strategies, students will eventually realize that it is valuable to know more than one method.

1. As a quick quiz, give students a list of 3-5 problems, and have them explain which problem-solving strategy could be used to solve each. For additional assessment, you could have students solve the problems using the strategies they suggested.

2. Have students keep a math journal in which they write about their understanding of this topic. For this lesson, students can respond to the following prompt: Choose four of the problem-solving strategies that we've considered. Compare and contrast the four strategies you've chosen; how are they similar, how are they different, and to what type of problem could each strategy be best applied?

3. Check students' math journals to be sure that they have correctly recorded descriptions for each of the problem-solving strategies that were discussed during this lesson.

4. Did each student successfully complete the Independent Practice Activity Sheet? Were the answers correct? Did students clearly explain their work?

5. Observe and evaluate students' participation in the pair and group discussions. You may wish to use the Kaplan 4-Point Rubric to score students in three areas: Solution Process, Presentation, and Understanding.

	Solution Process	Presentation	Understanding
1 **Unsatisfactory**	Not engaged in group's solution of the problem(s). Does not contribute to group effort at all.	Does not participate in the group presentation.	When asked, is unable to summarize the group's solution to the problem(s).
2 **Developing**	Is attentive while group solves problem(s) and contributes minimally to group effort.	Participates minimally in the group presentation.	Is able to explain parts of the group's solution, but indicates an incomplete understanding of how the steps in the solution led to the correct answer.
3 **Mastery**	Participates in group's solution of the problem(s). Contributes significantly by suggesting strategies, performing computation, and/or recording work.	Participates in the group presentation in two of the following ways: explaining strategies; showing calculations; drawing accompanying charts, lists, or diagrams; and/or answering questions from peers.	Is able to explain the group's solution completely. Explanation indicates an understanding of the mathematics behind the solution, with some gaps.
4 **Excellence**	Contributes significantly by suggesting strategies, performing computation, and/or recording work. Cooperates in the group by listening to all suggestions, and helps to evaluate all solution methods before selecting an approach.	Cooperates with group members in presenting the solution. Contributes significantly in three or more of the following ways: explaining strategies; showing calculations; drawing accompanying charts, lists, or diagrams; and/or answering questions from peers.	Is able to explain multiple solutions to the problem. Explanation indicates a full understanding of the mathematics behind the solution, with no gaps.

ENCORE! ENCORE!

EXTENSION ACTIVITIES

1. If more time is available, students will gain a deeper conceptual understanding of using multiple problem-solving strategies by completing the SkillVenture lesson.
2. As you proceed through your regular curriculum, look for opportunities to have students use multiple problem-solving strategies so that they continue to practice applying these skills in various scenarios.
3. Have students solve each of the problems on the Guided Practice Activity Sheet using a problem-solving strategy other than the one suggested.
4. Have each student create a problem that can be solved using at least two different strategies. In their math journals, the author of the problem can explain how various strategies could be used. The problems can then be exchanged and solved by other students.

Name ______________________________ Date ________________

Problem-Solving Strategies 1

GUIDED PRACTICE

Directions:

- Read each problem and think about how you might solve it.
- Then, read the description of how one student attempted to solve the problem and indicate the name of the strategy they used.
- Finally, solve the problem using the same strategy.

1. In an ultimate frisbee tournament with 8 teams, each team played every other team once. How many games were played?

   Attempted Solution:

   On a piece of paper, Giselle drew 8 dots, one for each team. She connected the dots with lines. Each line represented a game, so she counted the number of lines.

   Strategy Used? ________________________

2. There are 15 teams at a single-elimination tournament. “Single-elimination” means that a team plays until it loses one game to another team. How many games must be played to determine a champion?

   Attempted Solution:

   Kenneth used 15 pennies to represent the teams, and then he paired the pennies to represent two teams playing one another until a champion was determined. As he did this, Kenneth was counting the number of games that occurred.

   Strategy Used? ________________________

GUIDED PRACTICE *(cont'd)*

3. The width of an ultimate frisbee field is one-third its length. If the area of the field is 4,800 square yards, how wide is an ultimate frisbee field?

   Attempted Solution:

   Sarah assumed that the width was 20 yards, so the length must be 60 yards. The area would be 20×60, or 1,200 square yards. Since that was too small, Sarah tried larger numbers until she found the answer.

   Strategy Used? ______________________

4. For the local frisbee league, every player must have a uniform with a two-digit number. Jeremy had the following criteria for choosing his number:

   - It can only use the digits 1, 3, 4, 5, and 7.
   - It must be odd.
   - It has to be less than 50.
   - The sum of the digits must be greater than 10.

   Based on these criteria, what uniform number did Jeremy choose?

   Attempted Solution:

   Latoya made a list of the possible two-digit numbers, and crossed out the ones that didn't fit Jeremy's criteria.

   Strategy Used? ______________________

Name ______________________ Date ______________

5. The size of the local frisbee league grows every year. Two years ago, there were only 8 teams. Last year, there were 12 teams. This year, there are 18 teams. How many teams will the league have next year?

   Attempted Solution:

   Antonia noticed that the number of teams increased annually by 4, and then by 6, so she predicted the number of teams should increase by 8 next year.

   Strategy Used? ______________________

Name ______________________________ Date ______________

:: Problem-Solving Strategies 1

INDEPENDENT PRACTICE

For each problem below, indicate all of the strategies that could be used to solve it. Then, use one of the strategies to solve the problem.

- Consider a Simpler Case
- Eliminate
- Work Backwards
- Make a Model
- Guess and Check
- Look for a Pattern
- Make a Table, Chart, or List
- Draw a Diagram
- Use an Equation

1. An architect built a scale model of an office building. The model is 56 inches long and 20 inches wide. The actual building is 70 feet wide. How many feet long is the actual building?

   Strategy: ______________________

2. The combination of Martika's lock contains three numbers, each of which is an integer between 1 and 60. Martika has forgotten the last number, but she remembers that the first two are 19 and 27. She also remembers that the third number is greater than the sum of the other two, is odd, and contains the digit 3. What is the third number in Martika's locker combination?

   Strategy: ______________________

Name ______________________ Date ______________

INDEPENDENT PRACTICE *(cont'd)*

3. Axis rents a car for a base fee of $46, plus $23 per day. To rent the same car, Wertz charges a base fee of $18, plus $30 per day. The total rental cost will be equal at the two companies if the car is rented for how many days?

   Strategy: ______________________

4. Sixty percent of the members of the Elkhorn High School band are boys. One-third of the boys and one-half of the girls play percussion instruments. If there are 18 boys in the band, how many girls in the band play percussion?

   Strategy: ______________________

:: Problem-Solving Strategies 1

BEHIND THE SCENES ::
BACKGROUND INFORMATION

Students will use multiple problem-solving strategies.

1. Students should already be familiar with the following vocabulary: **equation.**
2. Materials:
   - student math journals
   - pencils
   - chalkboard/chalk
   - copies of Activity Sheets: Opening Activity Sheet and Independent Practice Activity Sheet

SETTING THE STAGE :: *approximately 6 minutes*
OPENING ACTIVITY

1. Distribute the Opening Activity Sheets as students enter the classroom.
2. Say, "Using a pencil, write one problem-solving strategy in each square of the grid on the Activity Sheet. To refresh your memory, discuss with a partner the various problem-solving strategies you've learned and used. If you're not able to fill in all 16 squares, that's okay. Just fill in as many as you can."
3. Allow students 2 minutes to fill in as many strategies as they can. Circulate around the room as students fill in their grids.
4. Call on students to share the strategies they listed on their grids. As students suggest strategies, create a list on the chalkboard or overhead projector. Ask students to briefly explain each strategy they suggest.
5. At a minimum, be sure that the following strategies have been suggested, along with a brief explanation of each:
   - **Act It Out:** With other students, perform the actions in the problem.
   - **Draw a Diagram:** Use a picture to visualize information that might be helpful in solving the problem.
   - **Eliminate:** Use the information in the problem to throw out answers that are impossible.
   - **Guess and Check:** Make a reasonable guess, and test it within the problem. If it doesn't work, revise your guess based on the result of your first try, and check again.

- **Make a List:** Create a list of answer possibilities based on the information given in the problem.
- **Make a Model:** Use tokens or other items to represent the pieces in the problem.
- **Make a Table, Chart, or Graph:** Use a table with multiple columns, a t-chart, or a coordinate graph to better understand the problem.
- **Solve a Simpler Problem:** Try to solve a similar, but less complicated problem to provide insight to the more difficult problem you are trying to solve.
- **Use an Equation:** Assign variables and create equations to solve the problem.
- **Use Logical Reasoning:** Think about the information given, and deduce the necessary information.

6. Once all strategies suggested by students have been listed, say, "Make sure you use each of these strategies at least once in your grid. Then, fill in the remaining squares with any strategies you'd want to use. The only rule is that you are not allowed to use any strategy more than three times. Be careful how you place the strategies, though, because you'll be using these grids for a game of *Problem-Solving Bingo*."

7. Allow students 1 minute to fill in their grids. Circulate around the room to make sure that all students have completed them.

DRESS REHEARSAL :: approximately 12 minutes

INSTRUCTION & GUIDED PRACTICE

1. Explain to students that they will be using the grids from the previous activity to play *Problem-Solving Bingo*. Say, "I will show you a problem, and you will choose a strategy that could be used to solve it."

2. Read the following rules to students:
   - Each problem that I show you will be indicated by a letter. When you decide on a strategy to solve the problem, mark the letter on your grid in a square that contains that strategy.
   - Most of the problems can be solved using more than one strategy; however, you may only use a problem once, so you will need to decide which strategy to use.
   - The object is to cover 4 squares in a row, in a column, or along a diagonal. When you do, yell, "Bingo!" and we'll check to make sure that you've applied the strategies correctly.

KAPtip Remind students that they are not to solve the problems at this time. They only need to select an appropriate problem-solving strategy.

3. Display the following problems, one at a time, on the chalkboard or overhead projector. Allow students 30 seconds per problem to decide which strategies could be used, and to mark their bingo cards.

KAPtip The following problems are not arranged in any particular order, and they may be presented to students in any order you wish. Play the game more than once by cutting the problems apart and randomly selecting one to read. In addition, you may choose to use problems from other sources to supplement the ones given here.

A. A bag contains four blue, four red, and four yellow marbles. How many blue marbles must be added so that the bag contains 75% blue marbles?

Possible Strategies:

- Draw a Diagram
- Use an Equation
- Make a Chart
- Guess and Check

B. In the year 2000, the number of people living on the Isles of Mynos was only 300. The population there triples every 15 years. Estimate the year in which the population will exceed 3,000.

Possible Strategies:

- Make a Chart
- Use an Equation
- Make a List
- Look for a Pattern

C. What is $1 + 2 - 3 + 4 - 5 + 6 - ... + 18 - 19 + 20$?

Possible Strategies:

- Look for a Pattern
- Solve a Simpler Problem
- Make a List

D. Twelve people sit in a circle. One person walks around the circle and taps every third person on the shoulder. A person tapped on the shoulder must then leave the circle. If the people in the circle are lettered A through L, and C is the first person to leave the circle, who will be the last person remaining?

Possible Strategies:

- Act it Out
- Draw a Diagram
- Look for a Pattern
- Make a List
- Eliminate
- Guess and Check

E. The Panthers are playing in a single-elimination tournament with seven other teams. Every team is equally likely to win any of the games they play. What is the probability that the Panthers will win the tournament?

Possible Strategies:

- Use a Formula
- Draw a Diagram

F. Four black and three white tokens are stacked one on top of another. How many different arrangements of these tokens are possible?

Possible Strategies:

- Draw a Diagram
- Make a Model
- Use a Formula
- Act it Out
- Solve a Simpler Problem

G. How much longer is 1% of an hour than 40% of a minute?

Possible Strategies:

- Use an Equation
- Draw a Diagram

H. What is the probability of rolling an even number on a standard die?

Possible Strategies:

- Make a List
- Use a Formula

I. A game for 2 to 4 players contains a bag of tokens. The number of tokens can always be divided evenly among the players. What is the least number of tokens in the bag?

Possible Strategies:

- Guess and Check
- Make a List
- Eliminate

J. Thirty students line up in a row. There are never more than 3 consecutive boys in the row. What is the greatest number of boys in the class?

Possible Strategies:

- Act it Out
- Eliminate
- Guess and Check
- Draw a Diagram
- Make a List
- Make a Model

KAPtip The strategies listed above are only suggestions. Students may propose other strategies, as well. They should be accepted if students can properly explain how the strategy would be applied.

4. When a student has called "Bingo!" have the class check his or her work by reviewing the problems and strategies that students used.

KAPtip Likely, more than one student will call bingo at the same time. Be sure to check each student's strategies to ensure that they have been correctly applied. In addition, discuss all possible strategies that could be used for each problem as they are reviewed.

5. If time permits and there are problems remaining, play another round of *Problem-Solving Bingo*.

KAPtipq You may wish to use a larger collection of problems for this game. Select items from state assessment tests, student textbooks, and other sources. In addition, you can vary the game by requiring students to cover all 16 squares instead of just covering one row, column, or diagonal.

Guided Practice Bingo Problem Solutions:

Although students are not required to solve the problems used in Problem-Solving Bingo, the following solutions are given in the event that you would like to have them do so. For each problem A to J, only one solution strategy is illustrated here, although additional strategies may work.

A) *Guess and Check*

If 10 blue marbles are added to the bag, the bag will contain 14 blue marbles and a total of 22 marbles, and the percentage of blue marbles will be $\frac{14}{22}$, or 63.6%. That's not enough for our condition, so more blue marbles are needed.

If 15 blue marbles are added, the bag will contain 19 blue marbles and 27 marbles total, which is $\frac{19}{27}$, or 70.3%. Still not enough.

If 20 blue marbles are added, the bag will contain 24 blue marbles and 32 marbles total, which is $\frac{24}{32}$, or 75%. That's the required percentage, so 20 blue marbles must be added to the bag.

B) *Make a Table*

Year	Population
2000	300
2015	900
2030	2,700
2045	8,100

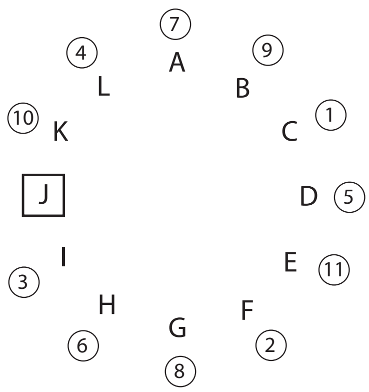
The population in Mynos is very close to 3,000 in the year 2030, so a reasonable estimate would be the year 2031.

C) *Look for a Pattern*

Notice that the sum can be rewritten as $1 + (2 - 3) + (4 - 5) + \ldots + (18 - 19) + 20$. In each parentheses, the value of the difference is -1, and there are 9 parentheses in all. So, the value of the expression is $1 + 9(-1) + 20$, or 12.

D) *Draw a Diagram*

Name the people in the circle letters A through L. The diagram below shows when each person is removed. Person J is the last one remaining.



E) *Use Logical Reasoning*

In an eight-team tournament, a team must win three games to become champion. Further, if each team has a probability of $\frac{1}{2}$ of winning a game, the probability that they win all three games is $(\frac{1}{2})^3$, or $\frac{1}{8}$.

F) *Use a Formula*

There are 7 positions into which each token can be placed. If the black tokens are placed first, then the positions of the white tokens will be determined. Consequently, choose 4 positions for the black tokens; there are ${}_7C_4$, or 35, ways to choose 4 of the 7 positions. The white tokens can then be placed in the 3 unoccupied positions. Hence, there are 35 possible arrangements.

G) *Use an Equation*

In seconds, 1% of an hour can be found with a proportion:

$\frac{1}{100} = \frac{x}{3600}$

$100x = 3600 \times 1$

$x = 36$ *seconds*

40% of a minute can be found using a proportion, as well:

$\frac{40}{100} = \frac{x}{60}$

$100x = 40 \times 60$

$x = 24$

So, 1% of an hour is 36 − 24, or 12 seconds longer than 40% of a minute.

H) *Make a List*

The three even numbers on a die are 2, 4, and 6. Because there are 6 numbers on a die in all, the probability of rolling an even number is $\frac{3}{6}$, or $\frac{1}{2}$.

I) *Make a Table*

Number of Tokens	*Does It Work?*	*Why or Why Not?*
4	*No*	*Not divisible by 3*
5	*No*	*Prime number*
6	*No*	*Not divisible by 4*
7	*No*	*Prime number*
8	*No*	*Not divisible by 3*
9	*No*	*Not divisible by 2 or 4*
10	*No*	*Not divisible by 3*
11	*No*	*Prime number*
12	*Yes*	*Divisible by 2, 3, and 4*

J) *Draw a Diagram*

The following diagram shows that there must be at least 7 girls in the class, so the maximum number of boys is 23.

BBB G BBB G BBB G BBB G BBB G BBB G BBB G BB

SHOW TIME! :: *approximately 22 minutes*

INDEPENDENT PRACTICE

1. Divide the class into groups of 3 or 4 students.
2. Say, "Okay, you've played Bingo using your card, and probably noticed that some strategies were easier to cover than others. At this time, you may reconfigure your board, but the same rules still apply. You must use each problem-solving strategy at least once, and you are not allowed to use any strategy more than three times."

KAPtip If necessary, provide additional copies of the Activity Sheet for students to create a new grid. Or, suggest that students create a new grid on a blank piece of paper.

3. Allow students to reconfigure their boards as you distribute the Independent Practice Activity Sheets.

4. Say, "Discuss each problem on the Activity Sheet with your group, and determine which strategies could be used to solve them. Then, just as you did in the Bingo game, use each problem once to cover a square on your bingo board with an appropriate strategy. The difference here is that you must cover every square on your grid, not just four squares in a row."

5. Answer any questions that students may have.

6. Explain that once each student has covered all of the squares on their grid, they are to solve four problems that are listed in one row, column, or diagonal. The strategy that was covered should be the one used to solve the problem. Point out that there is space at the end of the Independent Practice Activity Sheet where the four problems should be solved.

   KAPtip Time permitting, you may wish to have students share some of their solutions with the class.

7. Collect the Independent Practice Activity Sheets from students before the end of class.

   KAPtip Review the Activity Sheets to be sure that students were able to apply various problem-solving strategies.

Independent Practice Activity Sheet Answers:

A) Look for a Pattern; Eliminate; Make a List

When the tens digit is 1, there is only one number with a smaller units digit: 10

When the tens digit is 2, there are two numbers: 21 and 20

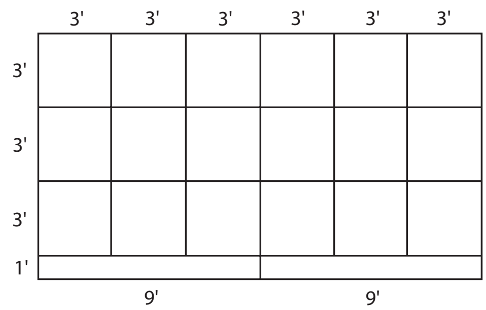
When the tens digit is 3, there are three numbers: 32, 31, and 30

When the tens digit is n, there are n numbers with a smaller units digit.

So, there are $1 + 2 + 3 + \ldots + 9$, or 45 two-digit numbers in which the tens digit is greater than the units digit.

B) Draw a Diagram; Use an Equation; Solve a Simpler Problem; Look for a Pattern; Guess and Check

A square yard is a square that measures 3 feet on a side; its area is 3 ft $\times$ 3 ft, or 9 ft^2. So, a square yard is also any region that has an area of 9 ft^2. The following diagram shows how a 10 ft $\times$ 18 ft area can be divided into 20 square yards:



Because the carpet costs $2.50 per square yard, the total cost will be $20 \times 2.50 = \$50$.

C) *Use an Equation; Make a List; Use Logical Reasoning*

The month of February has 28 days in a non-leap year, and 29 days in a leap year. Leap years occur every fourth year, except for years that are multiples of 400 (such as 1600, 2000, and 2400), which are not leap years. So, of every 400 years, 99 of them are leap years. Therefore, there are 99(29) + 301(28), or 11,299 days in February in the course of 400 years. On average, that's $\frac{11{,}299}{400}$, or about 28.25 days in February.

D) *Use an Equation; Look for a Pattern; Guess and Check*

We can begin with a guess that the price last year was $50. Consequently, it rose $4, which would be an increase of $\frac{4}{50}$, or 8.00%. That's too low, so we'll revise and try again.

Guess, instead, that the price last year was $49. That's an increase of $5, or $\frac{5}{49} = 10.20\%$. That's just a little too high. Revise and try again.

Try $49.10. That's an increase of $4.90, or $\frac{4.90}{49.10} = 9.98\%$. Still a little low, but we're getting closer.

A guess of $49.09 yields an increase of $\frac{4.91}{49.09} \approx 10.00\%$.

A similar guess-and-check process will yield that the price two years ago was $44.63, and the price three years ago was $40.57.

E) *Draw a Diagram; Guess and Check; Make a Table, Chart, or List; Use an Equation*

The following table shows the perimeter and area for a rectangle whose length is twice its width. The eighth row shows that the rectangle with a perimeter of 48 feet has an area of 128 square feet.

Width (feet)	Length (feet)	Perimeter (feet)	Area (square feet)
1	2	6	2
2	4	12	8
3	6	18	18
4	8	24	32
5	10	30	50
6	12	36	72
7	14	42	98
8	16	48	128
9	18	54	162
10	20	60	200

F) Make a List; Guess and Check; Eliminate; Look for a Pattern

There are only 12 pairs of positive numbers that have a sum of 25: (1,24), (2,23), (3,22), (4,21), (5,20), (6,19), (7,18), (8,17), (9,16), (10,15), (11,14), and (12,13). The products of the numbers in those pairs are 24, 46, 66, 84, and so on, respectively. From the pattern at the beginning of the list, it seems that the products increase, so it would also seem that the largest product would be $12 \times 13 = 156$. (To check, notice that $11 \times 14 = 154$.)

G) Make a List; Use an Equation or Formula; Look for a Pattern

The numbers between -4 and 9 are -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, and 8. There are 12 numbers in that list.

H) Draw a Diagram; Use an Equation; Guess and Check

A conversion can be used to solve this problem. There are 3,600 seconds in one hour, and 5,280 feet in a mile, so:

$$\frac{55 \text{ feet}}{22 \text{ seconds}} \times \frac{3{,}600 \text{ seconds}}{1 \text{ hour}} \times \frac{1 \text{ mile}}{5{,}280 \text{ feet}} \approx 1.705 \text{ miles per hour.}$$

I) Look for a Pattern; Make a Table, Chart, or List; Guess and Check; Solve a Simpler Problem; Use an Equation

The following table shows the total amount given when a $\frac{1}{4}$ cup measure is filled more than once. From the table, it seems that the cup must be filled 6 times to reach $1\frac{1}{2}$ cups.

Number of Times Filled	Total Amount
1	$\frac{1}{4}$ cup
2	$\frac{1}{2}$ cup
3	$\frac{3}{4}$ cup
4	1 cup
5	$1\frac{1}{4}$ cup
6	$1\frac{1}{2}$ cup
7	$1\frac{3}{4}$ cup
8	2 cups

J) Draw a Diagram; Use an Equation; Make a List; Make a Model; Solve a Simpler Problem; Look for a Pattern

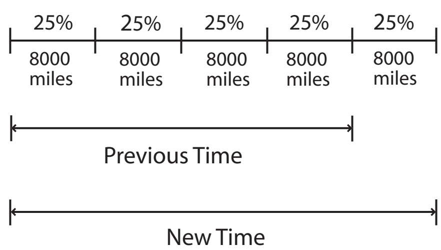
The following systematic, organized list shows every possible combination, where A, B, and C represent the roosters, and 1 and 2 represent the hens.

Pen 1	Pen 2
ABC1	2
ABC1	2
ABC2	1
AB12	C
AC12	B
BC12	A
ABC	12
AB1	C2
AC1	B2
BC1	A2
AB2	C1
AC2	B1
BC2	A1
A12	BC
B12	AC
C12	AB

From this list, you can see there are 16 ways that the hens and roosters could be divided.

K) *Use an Equation; Draw a Diagram; Guess and Check*

Because 25% is equal to $\frac{1}{4}$, the diagram shows 5 parts to this number line, each part equal to $\frac{1}{4}$ of the previous mileage. Because the previous tire is represented by 4 of these 5 segments, the answer is 32,000 miles.



L) *Use an Equation; Make a List; Look for a Pattern; Solve a Simpler Problem*

The ratio of blue paint to red paint is 3:4, but the ratio of blue paint to total paint in a mixture is 3:7. The following proportion can be used to solve the problem, with x representing the number of gallons of blue paint needed. (Note that the number of ounces in a gallon is not needed to determine an answer here, but it can be used to express the answer as ounces instead of gallons.)

$\frac{3}{7} = \frac{x}{3}$

$7x = 9$

$x = \frac{9}{7}$, *or approximately 1.29 gallons*

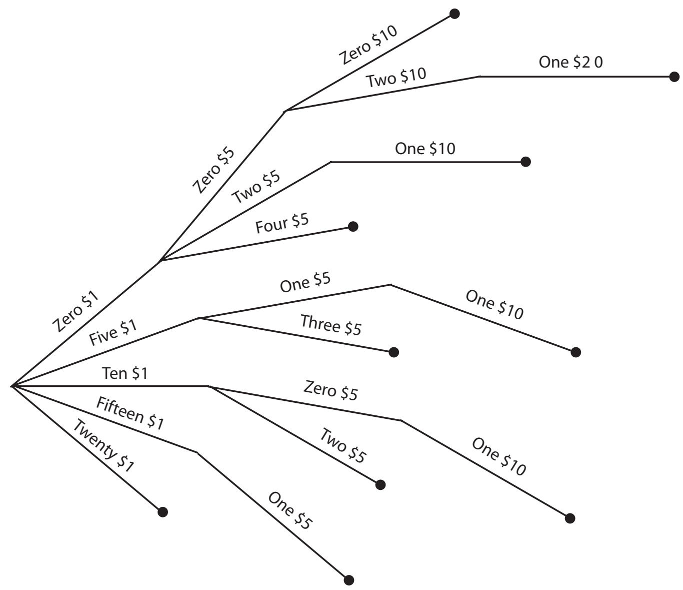
1.29 gallons is equal to 1.29×128, *or about 165 ounces*

M) *Guess and Check; Use Logical Reasoning; Eliminate; Make a List*

The greatest odd digit is 9, so the largest three-digit number with only odd digits must be 999.

N) *Make a List; Make a Model; Draw a Diagram*

The following tree diagram illustrates that there are 10 ways to have $20 with paper currency.



O) *Use an Equation; Guess and Check; Make a List; Use Logical Reasoning*

First, note that since the total ends in 5¢, there must be an odd number of quarters. (If there were an even number of quarters, the total would end in 0¢.)

So, begin with a guess that there are 5 quarters. That would yield 5(0.25) + 10(0.10) = $2.25. That's too low, so there must be more than 5 quarters.

Try 7 quarters. That yields 7(0.25) + 8(0.10) = $2.55, too low.

Try 9 quarters. That yields 9(0.25) + 6(0.10) = $2.85, correct.

Jeremy has 6 dimes.

P) *Look for a Pattern; Guess and Check; Use an Equation*

Assume that the set of five numbers is {1, 3, 5, 9, 17}. This set has a mean of $\frac{1+3+5+9+17}{5}$, *or 7.*

When 5 and 9 are removed, the remaining three numbers have a mean of $\frac{1+3+17}{3}$, *or 7. The mean of the removed numbers is* $\frac{5+9}{2}$, *or 7.*

Or, using an equation, assign variables a, b, c, d, and e as the five numbers. Because the mean of the five numbers is 7, then:

$\frac{a+b+c+d+e}{5} = 7$, *or* $a + b + c + d + e = 35$ *(1)*

If d and e are removed, then:

$\frac{a+b+c}{3} = 7$, *or* $a + b + c = 21$ *(2)*

Subtracting (2) from (1) leaves $d + e = 14$, *so the average of d and e, which is* $\frac{d+e}{2}$, *must be* $\frac{14}{2}$, *or 7.*

RAVE REVIEWS ::
ASSESSMENT

Throughout elementary school, students approach problems using a variety of methods. As they learn algebra and geometry, which often rely on formal methods, they tend to lose (or forget about) other problem-solving strategies. From time to time, it is valuable to remind students that strategies such as *Guess and Check* and *Draw a Diagram* are useful and reliable and, in fact, may be more efficient than more formal methods.

In addition, students may be uncomfortable using strategies for problems that they learned to solve a different way. Students may therefore resist the idea that other strategies are possible. By solving problems using a variety of strategies, students will eventually realize that it is valuable to know more than one method.

1. As a quick quiz, have students solve one problem using at least two different strategies.
2. Have students keep a math journal in which they write about their understanding of this topic. For this lesson, students can respond to the following prompt: It's important to know a variety of problem-solving strategies because __________ .
3. Check the Opening Activity Sheets. In particular, make sure that students have a sufficient list of problem-solving strategies as a result of the Opening Activity.
4. Did each student successfully complete the Independent Practice Activity Sheet? Were the answers correct? Did students clearly explain their work?
5. Observe and evaluate students' participation during the Independent Practice discussions. You may wish to use the Kaplan 4-Point Rubric (on next page) to score students in three areas: Solution Process, Presentation, and Understanding.

	Solution Process	Presentation	Understanding
1 **Unsatisfactory**	Not engaged in group's solution of the problem(s). Does not contribute to group effort at all.	Does not participate in the group presentation.	When asked, is unable to summarize the group's solution to the problem(s).
2 **Developing**	Is attentive while group solves problem(s) and contributes minimally to group effort.	Participates minimally in the group presentation.	Is able to explain parts of the group's solution, but indicates an incomplete understanding of how the steps in the solution led to the correct answer.
3 **Mastery**	Participates in group's solution of the problem(s). Contributes significantly by suggesting strategies, performing computation, and/or recording work.	Participates in the group presentation in two of the following ways: explaining strategies; showing calculations; drawing accompanying charts, lists, or diagrams; and/or answering questions from peers.	Is able to explain the group's solution completely. Explanation indicates an understanding of the mathematics behind the solution, with some gaps.
4 **Excellence**	Contributes significantly by suggesting strategies, performing computation, and/or recording work. Cooperates in the group by listening to all suggestions, and helps to evaluate all solution methods before selecting an approach.	Cooperates with group members in presenting the solution. Contributes significantly in three or more of the following ways: explaining strategies; showing calculations; drawing accompanying charts, lists, or diagrams; and/or answering questions from peers.	Is able to explain multiple solutions to the problem. Explanation indicates a full understanding of the mathematics behind the solution, with no gaps.

ENCORE! ENCORE! ::

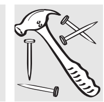
EXTENSION ACTIVITIES

1. As you proceed through your regular curriculum, look for opportunities to have students use multiple problem-solving strategies so that they continue to practice applying these skills in various scenarios.
2. Have students solve the problems from the Independent Practice Activity Sheet using as many strategies as possible.
3. Offer bonus points to any student who solves a problem using a problem-solving strategy other than those their classmates came up with.
4. Give problems to students, but instead of allowing them to solve the problem using whatever method they wish, require them to use a particular strategy. Then, ask students to check their work using a strategy of their choice.
5. Assign an Internet research project. Ask students to research problem-solving strategies. In addition to finding strategies that can be used for solving math problems, have them find strategies that scientists, businesspeople, engineers, and other professionals use to tackle problems in the workplace.

Name ______________________________ Date ______________

:: Problem-Solving Strategies 1

OPENING ACTIVITY



In each square of the grid below, list a problem-solving strategy that you have reviewed in class. With each strategy, include a brief description, including any notes you may need to remember how the strategy is applied.

Name ______________________ Date ______________

:: Problem-Solving Stategies 1

INDEPENDENT PRACTICE

With your group, discuss possible strategies to solve each of the following problems. Then, place the letter for each problem on a square with an appropriate strategy. Every square on your grid should be covered; if necessary, move problems that can be solved with more than one strategy to help cover your entire grid.

After your grid is covered, choose four problems along any row, column, or diagonal, and solve them using the strategy from each square.

A. For how many two-digit numbers is the tens digit larger than the units digit?

B. Alice's bedroom measures 10 feet by 18 feet. If carpet costs $2.50 per square yard, how much will carpet for Alice's room cost?

C. On average, how many days does the month of February have?

D. The value of a stock has risen 10% each year over the last 3 years. If the price of the stock is now $54, what was the price 3 years ago?

Name ______________________________ Date ____________________

INDEPENDENT PRACTICE *(cont'd)*

E. The length of a rectangle is twice its width. The perimeter of the rectangle is 48 feet. What is the area of the rectangle?

F. What is the largest possible product of two positive numbers that have a sum of 25?

G. How many integers are there between -4 and 9?

H. A boat floating down the Potomac River takes 22 seconds to pass under a bridge that is 55 feet wide. What is the speed of the river's current in miles per hour?

I. How many times must a $\frac{1}{4}$ cup measure be filled to make $1\frac{1}{2}$ cups?

J. In how many ways can three roosters and two hens be divided into two pens?

Name ______________________ Date ______________

INDEPENDENT PRACTICE *(cont'd)*

K. An ad reads, "Our new tires last 25% longer than our previous tires." If a new tire lasts 40,000 miles, how many miles did an old tire last?

L. Lindsey mixed 3 ounces of blue paint with 4 ounces of red paint to create a shade of purple to paint her room. If it will take 3 gallons to paint her entire room, how much blue paint will she need? (There are 128 ounces in a gallon.)

M. What is the largest three-digit number that contains only odd digits?

N. In how many different ways can you receive $20 using only paper money?

O. Jeremy has 15 coins in his bank. All of the coins are dimes or quarters, and the total value is $2.85. How many dimes does he have?

Name ______________________________ Date ____________________

INDEPENDENT PRACTICE *(cont'd)*

P. The average of five numbers is 7. When two numbers are removed, the average of the remaining three numbers is also 7. What is the mean of the removed numbers?

Solve the four problems along any row, column, or diagonal in your grid in the space provided below.

Problem: ____________________ Strategy: ____________________

Problem: ____________________ Strategy: ____________________

Problem: ____________________ Strategy: ____________________

Problem: ____________________ Strategy: ____________________